Exam. Code : 211003
Subject Code : 4865

M.Sc. (Mathematics) 3rd Semester

OPERATIONS RESEARCH—I

Paper : Math-578

Time Allowed—Three Hours] [Maximum Marks—100

Note :— Candidates are to attempt **FIVE** questions, **ONE** from each Section. **Fifth** question may be attempted from any Section. All questions carry equal marks. Non-programmable scientific calculator is allowed.

SECTION—A

I. (a) Use Big-M method to solve the following linear programming problem :

Maximize $Z = X_1 + 2X_2 + 3X_3 - X_4$

subject to the constraints

$$X_1 + 2X_2 + 3X_3 = 15$$

$$2X_1 + X_2 + 5X_3 = 20$$

$$X_1 + 2X_2 + X_3 + X_4 = 10$$

and $X_1, X_2, X_3, X_4 \geq 0$.

(b) Use the graphical method to solve the following linear programming problem :

Maximize $Z = 300X_1 + 400X_2$

subject to the constraints

$5X_1 + 4X_2 \leq 200$

$3X_1 + 5X_2 \leq 150$

$5X_1 + 4X_2 \geq 100$

$8X_1 + 4X_2 \geq 80$

and $X_1, X_2 \geq 0$.

(c) What is an unbounded solution, and how is this condition recognised in the graphical method ?

12+6+2=20

II. (a) Use two-phase simplex method to solve the following linear programming problem :

Maximize $Z = 3X_1 + 2X_2 + 2X_3$

subject to the constraints

$5X_1 + 7X_2 + 4X_3 \leq 7$

$-4X_1 + 7X_2 + 5X_3 \geq -2$

$3X_1 + 4X_2 - 6X_3 \geq 29/7$

and $X_1, X_2, X_3 \geq 0$.

(b) Find all the basic solutions of the following system, $X_1 + 2X_2 + X_3 = 4$, $2X_1 + X_2 + 5X_3 = 5$ and prove that they are non-degenerate.

(c) What is an infeasible solution, and how does it occur ? How is this condition recognised in the graphical method ? 12+6+2=20

SECTION—B

III. (a) If **x*** and **y*** be feasible solutions to the primal and dual linear programming (LP) problems, respectively, then show that a necessary and sufficient condition for **x*** and **y*** to be optimal solutions to their respective problem is, $y_i \cdot x_{n+i} = 0$, $i = 1, 2, 3, \ldots, m$ and $x_j \cdot y_{m+j} = 0$, $j = 1, 2, \ldots, n$ where x_{n+1} is the i-th slack variable in the primal LP problem and y_{m+j} is the j-th surplus variable for the dual LP problem.

(b) Consider the following transport problem. Suppose there are penalty costs for every unsatisfied demand unit which are given by 5, 3 and 2 for destination I, II and II, respectively. Find the optimal solution.

		To			
		I	II	III	Supply
From	A	5	1	7	10
	B	6	4	6	80
	C	3	2	5	15
Demand		75	20	50	

12+8=20

IV. (a) Find the solution of primary by solving the dual of the following linear programming problem :

Minimize $Z = 3X_1 - 2X_2 + 4X_3$

subject to the constraints

$3X_1 + 5X_2 + 4X_3 \geq 7$

$6X_1 + X_2 + 3X_3 \geq 4$

$7X_1 - 2X_2 - X_3 \leq 10$

$X_1 - 2X_2 + 5X_3 \geq 3$

$4X_1 + 7X_2 - 2X_3 \geq 2$

and $X_1, X_2, X_3 \geq 0$.

(b) Consider a firm having two factories. The firm is to ship its products from the factories to three retail stores. The number of units available at factories X and Y are 200 and 300, respectively, while those demanded at retail stores A, B and C are 100, 150 and 250 respectively. Rather than shipping the products directly from factories to retail stores, it is asked to investigate the possibility of trans-shipment. The transportation cost (in rupees) per unit is given in the table below. Find the optimal shipping schedule :

		Factory		Retail Store		
		X	**Y**	**A**	**B**	**C**
Factory	**X**	0	8	7	8	9
	Y	6	0	5	4	3
Retail Store	**A**	7	2	0	5	1
	B	1	5	1	0	4
	C	8	9	7	8	0

12+8=20

SECTION—C

V. (a) The airline company has drawn up a new flight schedule that involves five flights. To assist in allocating five pilots to the flight, it has asked them to state their preference scores by giving each flight a number out of 10. The highest the number, the greater the preference. A few of these flights are unsuitable to some pilots, owing to domestic reasons. These have been marked with '×'. What should be the allocation of the pilots to flights in order to meet as many preferences as possible ?

Flight Number

		I	II	III	IV	V
	A	8	2	×	5	4
	B	10	9	2	8	4
Pilot	C	5	4	9	6	×
	D	3	6	2	8	7
	E	5	6	10	4	3

(b) In a game of matching coins with two players, suppose A wins one unit of value when there are two heads, wins nothing when there are two tails and losses ½ unit of value when there is one head and one tail. Determine the payoff matrix, the best strategies for each player and the value of the game to A. 10+10=20

VI. (a) A salesman has to visit five cities A, B, C, D and E. The distances (in hundred kilometres) between the five cities are given below. If the salesman starts from city A and has to come back to city

A, which route should he select so that the total distance travelled is minimum ?

		To City				
		A	B	C	D	E
	A	–	1	6	8	4
	B	7	–	8	5	6
From City	C	6	8	–	9	7
	D	8	5	9	–	8
	E	4	6	7	8	–

(b) Solve the following game after reducing it to a 2×2 game :

	Player B-1	Player B-2	Player B-3
Player A-1	1	7	2
Player A-2	6	2	7
Player A-3	5	1	6

10+10=20

SECTION—D

VII. (a) Solve the integer linear programming problem (ILPP) using the cutting plane algorithm :

Maximize $Z = 2X_1 + 20X_2 - 10X_3$

subject to the constraints

$$20X_1 + 20X_2 + 4X_3 \le 15$$

$$6X_1 + 20X_2 + 4X_3 = 20$$

and $X_1, X_2, X_3 \ge 0$ and are integers.

(b) Determine the value of u_1, u_2 and u_3, so as to :

Maximize $Z = u_1 \cdot u_2 \cdot u_3$

subject to the constraints

$u_1 + u_2 + u_3 = 10$

and $u_1, u_2, u_3 \geq 0$. 10+10=20

VIII. (a) Solve the following (ILPP) using the branch and bound method :

Maximize $Z = 3X_1 + 5X_2$

subject to the constraints

$2X_1 + 4X_2 \leq 25$

$X_1 \leq 8$

$2X_2 \leq 10$

and $X_1, X_2 \geq 0$ are integers.

(b) Solve the following LPP by dynamic programming approach :

Maximize $Z = 8X_1 + 7X_2$

subject to the constraints

$2X_1 + X_2 \leq 8$

$5X_1 + 2X_2 \leq 15$

and $X_1, X_2 \geq 0$. 10+10=20